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BPS electromagnetic waves on giant gravitons

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Abstract: We find new $\frac{1}{8}$-BPS giant graviton solutions in $AdS_5 \times S^5$, carrying three angular momenta along S^5, and investigate their properties. Especially, we show that nonzero worldvolume gauge fields are admitted preserving supersymmetry. These gauge field modes can be viewed as electromagnetic waves along the compact D3 brane, whose Poynting vector contributes to the BPS angular momenta. We also analyze the (nearly-)spherical giant gravitons with worldvolume gauge fields in detail. Expressing the S^3 in Hopf fibration (S^1 fibred over S^2), the wave propagates along the S^1 fiber.

Keywords: AdS-CFT and dS-CFT Correspondence, D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction and conclusion

1/2 BPS gravitons with high angular momentum in $AdS_p \times S^q$ are shown to be expanded into higher dimensional brane objects, the so-called giant gravitons, by McGreevy, Susskind and Toumbas [1]. The massless gravitons get polarized to branes due to the Myers dielectric effect [2]. These giant gravitons can be BPS objects [3, 4], and also have been studied in relation to the AdS-CFT correspondence.

Less supersymmetric, 1/4 and 1/8 BPS, giant gravitons have also been found. Giant gravitons can be considered as finite D-branes, which can be studied by the worldvolume theory defined by the Dirac-Born-Infeld(DBI) and Chern-Simons(CS) actions. Especially supersymmetric giant gravitons extended in S^q were characterized by the intersection of S^q and a holomorphic surface by Mikhailov [5]. See also [6, 7]. Giant graviton as an extended D-brane could in principle carry dynamical world-volume gauge field, still keeping supersymmetry. [8, 9] treat some related objects, corresponding to strings ending on/dissolved into giant gravitons. In this work we show that this is possible and find some exact properties of the BPS gauge field living on the giant graviton of the type-IIB string theory on $AdS_5 \times S^5$, which contributes to both energy and angular momentum on S^5. Especially we find the explicit 1/8 BPS smooth electromagnetic wave solutions on a nearly-spherical BPS D3 brane, and study their quantum physics. (We will not treat the dual giant gravitons [3, 4] in this paper.)

While the general electromagnetic wave propagating on a three manifold would be very complicated, we find that the supersymmetric one is considerably simpler. This solution of course satisfies Gauss-Bianchi constraints. Especially when the world volume geometry of a giant graviton is S^3, which can be regarded as a Hopf-bundle of S^1 over S^2, we find explicit smooth configuration for all BPS electromagnetic waves. While the bundle structure is what makes such wave possible, roughly one can see that electric and magnetic fields are mutually orthogonal but of the same magnitude on S^2 and propagate along the S^1 fibre, so that the Poynting vector density is along the S^1 direction at each point. Such configuration turns out to be smooth and has finite energy. (As for the previous work for the study of fluctuations around spherical giant gravitons, see [10].) When a 1/8 BPS giant graviton is moving along S^5 with three angular momentum, such an electromagnetic wave on world volume seems to be still possible, preserving the same 1/8 supersymmetries.

For our giant gravitons, conserved quantities are three angular momenta, say, J_{12}, J_{34}, J_{56} along the S^5. Thus, such 1/8 BPS electromagnetic wave contributes to the BPS energy and angular momentum, which leads to additional degeneracy of the 1/8 giant graviton quantum states. Here we quantize all BPS electromagnetic waves on S^3, leading to quantized angular momentum contributions.

One immediate question is whether one can obtain more explicit solutions for the gauge fields when the shape of giant graviton is more complicated. In this paper we constructed explicit solutions for the nearly-spherical case. Presumably, 1/8 BPS giant gravitons can have more complicated topology, like three torus and so on, which may allow also 1/8 BPS electromagnetic wave. (Topologically nontrivial giant gravitons are constructed in the maximally supersymmetric plane wave background of M-theory [12, 13].) In addition, there could be also nontrivial gauge holonomy and/or flux along non-contractible cycle. It would be interesting to find such solutions explicitly. In somewhat different direction, there is some work on giant gravitons with nonzero gauge fields on the plane wave background obtained from the Penrose limit of $AdS_5 \times S^5$ [11]. Our work could be generalized to the plane wave case and shed some light on the subject.

Quantum mechanically, there is an enormous degeneracy of giant gravitons with given angular momenta, which could be countable in principle. When gravitational back reaction is included, such BPS object in AdS space does not appear as an extremal black hole, but as a 'superstar' with null or time-like naked singularity [14]. See also [15]. Recently, regular solutions of the 10 dimensional supergravity with one angular momentum has been studied [16], and there also has been some study of non-supersymmetric black holes carrying more than one charges [17]. However, the complete understanding of the quantum degeneracy of giant graviton states and the counting of these seems to be somewhat wanting.

A new class of extremal black hole solutions have been found in AdS space [18]. Besides angular momentum along S^5, they carry angular momentum also in the AdS_5 part. This solution has singularities cloaked inside a horizon with nonzero area. Thus it would be interesting to find BPS giant gravitons, with worldvolume electromagnetic wave, which carries angular momenta in the AdS_5 part also. $\frac{1}{2}$-BPS solutions of this type exist in the reference [19, 20].

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The gauge field solutions found in this paper prompt us to consider the $\frac{1}{8}$-BPS giant gravitons in $AdS_4 \times S^7$ made of $M5$ branes with four angular momenta [5], including the self-dual three form tensor field strength on the worldvolume. It is naturally conceivable that one needs to consider S^5 as S^1 fibration over $\mathbb{CP}^2$ [21]. In a related maximally supersymmetric plane wave background, BPS tensor modes around $\frac{1}{2}$-BPS vacuum have been observed [22]. It would be desirable to have clear geometric understanding as we got through the work of this paper.

In the matrix theory context, interesting observations have been made through a series of papers by Janssen *et.al.* [23, 24], which we think is somewhat related to our present work as well as future projects. They constructed $\frac{1}{2}$-BPS spherical giant gravitons in the $AdS_5 \times S^5$ and $AdS_4 \times S^7$ cases from the relevant matrix theories. The S^1 fibrations over suitable projective spaces are considered (taking advantage of fuzzy $\mathbb{CP}^1 = S^1$ and $\mathbb{CP}^2$) to form S^3 and S^5 giant gravitons.

The organization of this paper is as follows. In section 2 we review the construction of $\frac{1}{8}$-BPS giant gravitons without turning on world volume gauge fields. In section 3 we show that gauge fields can be turned on in a supersymmetric way. First we provide the general condition for the gauge fields to preserve $\frac{1}{8}$ supersymmetry. Then we consider the Gauss law and related constraints which should be further satisfied. We also compute the energy and angular momenta on S^5 carried by these configurations, and show that energy saturates the BPS bound given by sum of three angular momenta. In section 4 we provide the exact solutions of the constraint equations for gauge field on a spherical giant graviton and show that they are electromagnetic waves propagating along closed circles in Hopf fibration of S^3. We also quantize this explicit solution, assuming small fluctuations, and identify the angular momentum quanta of these modes. One appendix is included to explain technical facts.

2. $\frac{1}{8}$-BPS giant gravitons from holomorphic surfaces

In this section we review the giant graviton solutions without turning on worldvolume gauge fields. We will also clarify our notations and conventions.

2.1 Supersymmetry of $AdS_5 \times S^5$ background

We start by embedding $AdS_5 \times S^5$ with radii R into a 12 dimensional space $\mathbb{R}^{2+4} \times \mathbb{R}^6$, which would be useful throughout this paper. Writing the two radial coordinates of $\mathbb{R}^{2+4}$ and $\mathbb{R}^6$ as r_1 and r_2, respectively, the metric on $AdS_5 \times S^5$ is inherited from that of the flat space in a manifest way:

$$ds^2_{\mathbb{R}^{2+4}} = -R^2 dr_1^2 + r_1^2 ds^2_{AdS_5}\,, \qquad ds^2_{\mathbb{R}^6} = R^2 dr_2^2 + r_2^2 ds^2_{S^5} \tag{2.1}$$

with restriction to $r_1 = r_2 = 1$ subspace. The spin connection components containing r_1 or r_2 indices are as follows (characters with caret are local orthonormal frame indices):

$$\omega^{\hat{\mu}\hat{r}_1} = -\frac{1}{R}\, e^{\hat{\mu}}\,, \qquad \omega^{\hat{i}\hat{r}_2} = \frac{1}{R}\, e^{\hat{i}} \tag{2.2}$$

where μ and i denote five AdS_5 and S^5 indices, respectively, in appropriate coordinates, and $e^{\hat{a}}$ is the vielbein 1-form.

We summarize the construction of 32 Killing spinors in $AdS_5 \times S^5$ with self-dual 5-form fluxes, starting from 12 dimensional covariantly constant spinors. The Killing spinors should leave the IIB gravitino invariant under the following supersymmetry transformation

$$\delta\psi_M = \frac{1}{\kappa} D_M \epsilon + \frac{i}{4\kappa \cdot 480} F^{(5)}_{NPQRS} \Gamma^{NPQRS} \Gamma_M \epsilon = 0 \tag{2.3}$$

where $D_M\epsilon = \partial_M \epsilon + \frac{1}{4}\omega_M^{\ \hat{P}\hat{Q}} \Gamma_{\hat{P}\hat{Q}}\epsilon$. The IIB chirality condition is

$$\Gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}} \Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}} \epsilon = +\epsilon \,, \tag{2.4}$$

where $0 \sim 4/5 \sim 9$ are AdS_5/S^5 indices, respectively. The flux part is given as

$$\frac{1}{480} F^{(5)}_{NPQRS} \Gamma^{NPQRS} = \frac{1}{R}(\Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}} + \Gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}}) \stackrel{\text{eff}}{=} \frac{2}{R}\Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}} \tag{2.5}$$

when acting on antichiral spinors, like $\Gamma_M \epsilon$ in (2.3).

To solve this Killing spinor equation, we start from a 12 dimensional Dirac spinor Ψ, which has $2^6 = 64$ complex components. We will assume the Majorana representation with real gamma matrices. We require it to be covariantly constant in 12 dimensional sense. In the most trivial frame for the 12 dimensional vielbein, Ψ is simply a constant spinor since $\mathbb{R}^{2+4} \times \mathbb{R}^6$ is flat. However, performing a local Lorentz transformation to make $\Gamma^{\hat{0}} \sim \Gamma^{\hat{9}}$ and $\Gamma^{\hat{r}_1}$, $\Gamma^{\hat{r}_2}$ as numerical matrices, Ψ gains nontrivial dependence on the $AdS_5 \times S^5$ coordinates (but not on r_1 or r_2). Let us consider Ψ in this frame with the following two projection constraints (they are numerical projectors in both frames)

$$\Gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}\hat{r}_1} \Psi = -i\Psi \,, \qquad \Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}\hat{r}_2} \Psi = +i\Psi \,. \tag{2.6}$$

Our convention for gamma matrices is $(\Gamma^{\hat{0}})^2 = (\Gamma^{\hat{r}_1})^2 = -1$, while all the others square to 1. Then, using the expression (2.2), the 12 dimensional covariant constancy condition is rephrased in terms of $AdS_5 \times S^5$ coordinates as

$$\begin{aligned} 0 &= D_\mu \Psi + \frac{1}{2}\omega_\mu^{\ \hat{\nu}\hat{r}_1} \Gamma_{\hat{\nu}\hat{r}_1} \Psi = D_\mu \Psi + \frac{i}{2R} \Gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}} \Gamma_\mu \Psi \\ 0 &= D_i \Psi + \frac{1}{2}\omega_i^{\ \hat{j}\hat{r}_2} \Gamma_{\hat{j}\hat{r}_2} \Psi = D_i \Psi + \frac{i}{2R} \Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}} \Gamma_i \Psi \end{aligned} \tag{2.7}$$

which is nearly, but not exactly yet, the Killing spinor equation (2.3) in $AdS_5 \times S^5$ with the flux (2.5). To complete the construction, we have to make sure that IIB chirality condition (2.4) is satisfied. Ψ does not satisfy this condition, but the projected spinor

$$\epsilon \equiv \frac{1 - \Gamma^{\hat{r}_1\hat{r}_2}}{2} \Psi \,, \qquad \Gamma^{\hat{r}_1\hat{r}_2} \epsilon = -\epsilon \tag{2.8}$$

does. Since this projector commutes with all the matrices appearing in (2.7), we can make ϵ satisfy the same equation. However, after replacing Ψ by ϵ, $\Gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}}$ in (2.7) can be replaced by $\Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}}$, and we finally get the desired Killing spinor equation. As a 12 dimensional spinor, the final answer ϵ is subject to two projection conditions (2.4) and (2.8). Therefore, it carries 16 complex components, as required for the $AdS_5 \times S^5$ Killing spinor.

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2.2 $\frac{1}{8}$-BPS giant gravitons

In this subsection we review the D3 giant gravitons preserving $\frac{1}{8}$ supersymmetry and carrying three components of SO(6) angular momentum, using holomorphic surfaces [5]. We will present the details since it will be useful in the next section.

The D3 brane we are interested in stays at the origin of AdS_5 and has nontrivial shape and time evolution in S^5. It is constructed by the following procedure:

1. Regarding the embedding space $\mathbb{R}^6$ as $\mathbb{C}^3$ with holomorphic coordinates Z_1, Z_2, Z_3, consider any holomorphic surface given by an equation of the form $F(Z_1, Z_2, Z_3) = 0$.

2. Let us call Σ the 3-manifold given by the intersection of the above surface and the sphere $|Z_1|^2+|Z_1|^2+|Z_1|^2 = 1$. This is the D3 brane configuration at a given time, say $t = 0$, where t is the worldvolume time coordinate. We partially fix the worldvolume diffeomorphism by setting Rt to be the proper time for an observer sitting at the origin of AdS_5.

3. Given the above initial configuration, the time evolution of the brane is given as follows. The coordinate Z_k of a point on the 3-manifold evolves as $\dot{Z}_k = iZ_k$, where dot denotes t derivative. The worldvolume trajectory of the D3 brane is thus given as $F(e^{-it}Z_1, e^{-it}Z_2, e^{-it}Z_3) = 0$.

In the rest of this subsection we will summarize the proof that this configuration preserves $\frac{1}{8}$ supersymmetry.

To simplify the proof, let us follow [5] and introduce some useful notations. Let $\mathbf{e}^{\perp}$ be a unit vector in $\mathbb{R}^6$ which is normal to S^5. The time evolution $\dot{Z}_k = iZ_k$ can be phrased in a different way that the velocity vector is $I.\ \mathbf{e}^{\perp}$ at each point of D3 brane, where the operation I gives the complex structure to $\mathbb{R}^6$.[1] However, the vector $I.\ \mathbf{e}^{\perp}$ is not orthogonal to the spatial D3 manifold Σ, so it is not the physical velocity. We introduce another unit vector $\mathbf{e}^{\phi} \in T(S^5)$, aligned toward the direction of transverse velocity. Another unit vector in $T(S^5)$ transverse to Σ and normal to $\mathbf{e}^{\phi}$ is called $\mathbf{e}^{n}$. One can easily see that

$$I.\ \mathbf{e}^{\phi} = -\cos\alpha\ \mathbf{e}^{\perp} + \sin\alpha\ \mathbf{e}^{n}\,, \tag{2.9}$$

and that $\cos\alpha = \mathbf{e}^{\phi}\cdot I.\ \mathbf{e}^{\perp} \equiv v$ is the transverse velocity of the D3 brane in suitable worldvolume orientation. See appendix A for the proof.

The supersymmetry preserved by above configuration can be checked by investigating the supersymmetry plus compensating kappa symmetry transformation. For the D3 brane, it is given as follows (we follow the notation of the second reference in [25]):

$$\Gamma\epsilon = \epsilon \tag{2.10}$$

[1] Writing the components as $Z_k = X_k + iY_k$, and corresponding unit vectors as $\hat{x}_k$ and $\hat{y}_k$, we have $I.\hat{x}_k = \hat{y}_k$ and $I.\hat{y}_k = -\hat{x}_k$.

where ϵ is the Killing spinor obtained in the previous subsection, and

$$\Gamma = \frac{1}{\sqrt{\det(1+Y)}}\left[i\sigma_2 \otimes \left(1+\frac{1}{8}\gamma^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)\Gamma_{(0)} - \sigma_1 \otimes \left(\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu}\right)\Gamma_{(0)}\right]$$

$$Y^{\mu}{}_{\nu} \equiv g^{\mu\rho}F_{\rho\nu}\,, \qquad \Gamma_{(0)} \equiv \frac{1}{4!\sqrt{-\det g}}\,\epsilon^{\mu\nu\rho\sigma}\gamma_{\mu\nu\rho\sigma} \qquad ((\Gamma_{(0)})^2=-1)$$

$$\epsilon_{0123} = -\epsilon^{0123} = +1\,, \qquad \gamma_\mu = \Gamma_i\frac{\partial X^i}{\partial\sigma^\mu}\ :\ \ \text{induced gamma matrix}\,. \tag{2.11}$$

The 2×2 Pauli matrices act on the $\mathrm{SL}(2,\mathbb{R})$ indices of the type-IIB spinors. Since we are using the complex Killing spinor, they act as [26]

$$i\sigma_2\epsilon = -i\epsilon\,, \qquad \sigma_1\epsilon = i\epsilon^*\,, \qquad \sigma_3\epsilon = \epsilon^*\,. \tag{2.12}$$

We will consider the inclusion of gauge field $F_{\mu\nu}$ in the next section.

Without the gauge fields, the projection operator becomes

$$\Gamma = -i\Gamma_{(0)} = \frac{-i}{\sqrt{1-v^2}}(\Gamma^{\hat{0}} - v\Gamma^{\hat{\phi}})\hat{\Sigma} \tag{2.13}$$

where $\Gamma^{\hat{\phi}} = \Gamma(\mathbf{e}^\phi)$ is the Gamma matrix along $\mathbf{e}^\phi$ direction which squares to 1, and $\hat{\Sigma}$ is the product of three Gamma matrices corresponding to the spatial part of D3 worldvolume satisfying $\hat{\Sigma}^2 = -1$. Note that $\Gamma^{\hat{\phi}}$ and $\hat{\Sigma}$ anticommute since $\mathbf{e}^\phi$ is transverse to the D3 brane. Using the identification $v = \cos\alpha$ and the relation (2.9), the supersymmetry condition (2.10) can be written as

$$0 = (1-\Gamma^{\hat{r}_1\hat{r}_2})\Gamma^{\hat{0}\hat{r}_1\hat{n}\hat{\phi}}\left[1-\Gamma^{\hat{0}\hat{r}_1}\Gamma(\mathbf{e}^\phi)\Gamma(I.\ \mathbf{e}^\phi)\right]\Psi \ \rightarrow\ \Gamma^{\hat{0}\hat{r}_1}\Gamma(\mathbf{e}^\phi)\Gamma(I.\ \mathbf{e}^\phi)\Psi = +\Psi\,. \tag{2.14}$$

In obtaining this condition, we used the orientation convention

$$\hat{\Sigma}\Gamma^{\hat{n}\hat{\phi}\hat{r}_2}\Psi = \Gamma^{\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}\hat{r}_2} = i\Psi \tag{2.15}$$

together with the second condition of (2.6). We will work in the trivial frame where Ψ is a constant spinor. The solution for (2.14) in generic case is obtained as follows. First, (2.14) is solved by imposing[2]

$$\Gamma^{\hat{0}\hat{r}_1}\Psi = +i\Psi\,, \qquad \Gamma(\mathbf{e}^\phi)\Gamma(I.\ \mathbf{e}^\phi)\Psi = -i\Psi\,. \tag{2.16}$$

Note that the first projector is a numerical matrix also in the trivial frame, since the D3 brane is sitting at the origin of AdS_5. Writing the latter projector as

$$\Gamma\left(\frac{\mathbf{e}^\phi + iI.\ \mathbf{e}^\phi}{2}\right)\Gamma\left(\frac{\mathbf{e}^\phi - iI.\ \mathbf{e}^\phi}{2}\right) = \frac{1-i\Gamma(\mathbf{e}^\phi)\Gamma(I.\ \mathbf{e}^\phi)}{2}\,, \tag{2.17}$$

we are led to the nontrivial requirement

$$\Gamma\left(\frac{\mathbf{e}^\phi - iI.\ \mathbf{e}^\phi}{2}\right)\Psi = 0\,. \tag{2.18}$$

[2]Of course the other sign choice also solves (2.14). However, it turns out that only one of these two is compatible with (2.6).

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Since this matrix generically depends on all the S^5 coordinates, the only way to fulfill the requirement is to set

$$\Gamma(\partial_{Z_k})\Psi = \frac{1}{2}\left(\Gamma_{X_k} - i\Gamma_{Y_k}\right)\Psi = 0 \tag{2.19}$$

for all pair indices $k = 1, 2, 3$. These three projections are not independent: one is given by the other two using the condition (2.6). Therefore, two of these three together with the first of (2.16) make this configuration $\frac{1}{8}$-BPS.

3. Giant gravitons with worldvolume gauge fields

Having reviewed the giant gravitons without worldvolume gauge fields, we now turn to the generalization with gauge fields turned on.

3.1 Supersymmetry

We first set our notation for the induced metric on the worldvolume. It can be written as

$$g_{\mu\nu} = R^2 \begin{pmatrix} -1+v^2 & 0 \\ 0 & h_{ij} \end{pmatrix}, \qquad h_{ij} = e_i^{\hat{k}} e_j^{\hat{k}}\,, \tag{3.1}$$

where $i, j, k = 1, 2, 3$. The absence of g_{0i} components may be understood as being killed by time-dependent diffeomorphism. The quantity $e_j^{\hat{i}}$ is the spatial vielbein on D3. Its inverse is written as $e_{\hat{j}}^{i}$, satisfying $e_k^{\hat{i}}\ e_{\hat{j}}^{k} = \delta_{\hat{j}}^{\hat{i}}$, etc. The vielbein should not be confused either with the bulk vielbein used in the previous section, or with the various unit vectors written in bold characters. We are also going to specify a convenient expression for the spatial worldvolume metric h_{ij} in (3.1). On the 3 manifold Σ given by holomorphic surface as in the previous section, there exists an I-invariant sub-plane $T_0\Sigma$ in the tangent space at each point: let us call the unit vector (normalized by induced metric on Σ) normal to this plane as $\mathbf{e}^{\psi}$, following [5]. One can easily see from the definition that

$$I.\mathbf{e}^{\perp} = \sin\alpha\ \mathbf{e}^{\psi} + \cos\alpha\,\mathbf{e}^{\phi}\,. \tag{3.2}$$

At a given moment of time (say $t = 0$), we can choose one of our spatial coordinate as ψ such that its associated tangent vector ∂_ψ is proportional to $\mathbf{e}^{\psi}$. Then, one can write the general metric as follows:

$$ds_\Sigma^2 = h(x,\psi)\left(d\psi + \sum_{a=1,2} V_a(x,\psi)dx^a\right)^2 + g(x,\psi)\sum_{a=1,2}(dx^a)^2\,. \tag{3.3}$$

We took advantage of x^1-x^2 diffeomorphism to go to a sort of conformal gauge and have a common factor $g(x, \psi)$. The vielbein components are given as follows:

$$\begin{aligned} e^{\hat{\psi}} &= \sqrt{h}(d\psi + V_a dx^a)\,, \qquad e^{\hat{a}} = \sqrt{g}dx^a \\ e_{\hat{\psi}} &= \frac{1}{\sqrt{h}}\partial_\psi\,, \qquad e_{\hat{a}} = \frac{1}{\sqrt{g}}\left(\partial_a - V_a\partial_\psi\right), \end{aligned} \tag{3.4}$$

which should not be confused with the bulk vielbein we used in section 2. The choice of inverse vielbein $e_{\hat{\psi}}$ is indeed proportional to ∂_ψ, as we required.

The electric and magnetic fields are defined as

$$E_{\hat{i}} = \frac{1}{\sqrt{1-v^2}} f^i_{\hat{j}} F_{0j}\,, \qquad B_{\hat{i}} = \frac{1}{2}\epsilon_{ijk} f^l_{\hat{j}} f^m_{\hat{k}} F_{lm} \qquad (\epsilon_{123}=1)\,. \tag{3.5}$$

The square root factor is introduced for convenience. We will use the rescaled field strength $F_{\mu\nu} = R^2 F^{(scaled)}_{\mu\nu}$ in order not to have the R^2 factors here and there. With this convention, we compute various $F_{\mu\nu}$-dependent quantities appearing in (2.11). The relevant quantities are expressed as

$$\begin{aligned} \det(1+Y) &= 1 + |\vec{B}|^2 - |\vec{E}|^2 - (\vec{E}\cdot\vec{B})^2\ , \\ \frac{1}{8}\gamma^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} &= (\vec{E}\cdot\vec{B})\ \Gamma_{(0)}\,, \\ \frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} = \gamma^{\hat{t}\hat{i}}E_{\hat{i}} + \frac{1}{2}\epsilon_{ijk}\gamma^{\hat{i}\hat{j}}B_{\hat{k}} &= \frac{1}{\sqrt{1-v^2}}(\Gamma^{\hat{0}} - v\Gamma^{\hat{\phi}})(\vec{\gamma}\cdot\vec{E}) + \hat{\Sigma}(\vec{\gamma}\cdot\vec{B})\,. \end{aligned} \tag{3.6}$$

where the vectors denote spatial 3-vectors with indices expressed in local orthonormal frame on the worldvolume, and we used $\epsilon_{\hat{1}\hat{2}\hat{\psi}} = 1$. We also note that the sign convention for the tensor ϵ_{ijk} on Σ should be $\epsilon^{\hat{x}\hat{y}\hat{\psi}} = 1$, where x, y are the indices parametrizing the 2 manifold and become 'x-like' and 'y-like' variables, respectively, after being push-forwarded: that is, $I.e_{\hat{1}} = e_{\hat{2}}$. This can be shown by a careful sign check using the convention (2.15) and the I operation rules (2.9), (3.2).

Since our main motivation is looking for the states preserving the *same supersymmetry*, we require the relation

$$-i\Gamma_{(0)}\epsilon = \epsilon \ \rightarrow\ \frac{1}{\sqrt{1-v^2}}(\Gamma^{\hat{0}} - v\Gamma^{\hat{\phi}})\epsilon = i\hat{\Sigma}\epsilon\,, \tag{3.7}$$

or equivalently, (2.19) that we developed in the previous section. Therefore, we still have the same solution for the shape and its time evolution given by holomorphic surfaces. With (3.7) assumed, the supersymmetry condition (2.10) becomes

$$\frac{1}{\sqrt{1+|\vec{B}|^2 - |\vec{E}|^2 - (\vec{E}\cdot\vec{B})^2}}\left[1 + i(\vec{E}\cdot\vec{B}) - (\sigma_1)\otimes\hat{\Sigma}\vec{\gamma}\cdot(\vec{E}+i\vec{B})\right]\epsilon = \epsilon\,, \tag{3.8}$$

where σ_1 is understood to act on the whole complex quantity $(\vec{E}+i\vec{B})\epsilon$.

First of all, since the action of σ_1 is complex conjugation on ϵ (or $(\vec{E}+i\vec{B})\epsilon$), it is hard to expect supersymmetry if this term does not vanish.[3] Therefore we require

$$\vec{\gamma}\cdot(\vec{E}+i\vec{B})\epsilon = 0\,. \tag{3.9}$$

Then, looking at the remaining terms in (3.8), we should also require

$$\vec{E}\cdot\vec{B} = 0\,, \qquad |\vec{E}| = |\vec{B}| \tag{3.10}$$

to have supersymmetry.

[3] At the end of this subsection, we will show this term should vanish indeed to have supersymmetry.

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We now present some useful facts to solve (3.9) and (3.10) using the $\frac{1}{8}$ supersymmetry condition (2.19). At each point x on Σ, recall that there is a vector field $\mathbf{e}^{\psi}$ which does not close to $T\Sigma$ under the action of I, and a two dimensional subspace $T_0\Sigma$ orthogonal to $\mathbf{e}^{\psi}$ which is closed under I [5]. With our coordinate and vielbein choice (3.3) and (3.4), one obtains

$$\gamma_{\hat{1}} - i\gamma_{\hat{2}} = \Gamma^A \frac{\partial X^A}{\partial \sigma^k}\,(e^k_{\hat{1}} - ie^k_{\hat{2}}) \equiv \Gamma\left(e_{\hat{1}} - ie_{\hat{2}}\right) \tag{3.11}$$

where $e_{\hat{a}}$ $(a = 1, 2)$ are understood as push-forwards of the worldvolume vectors $e^i_{\hat{a}}$. Since $e_{\hat{a}}$ are the two orthonormal vectors spanning $T_0\Sigma$, we have

$$(e_{\hat{2}})^A = \left(I.\ e_{\hat{1}}\right)^A \ \rightarrow\ \gamma_{\hat{1}} - i\gamma_{\hat{2}} = \Gamma\left(e_{\hat{1}} - iI.e_{\hat{1}}\right). \tag{3.12}$$

We used the fact that vectors $e_{\hat{1}}$ and $e_{\hat{2}}$ behaves respectively as 'x' and 'y' direction after being push-forwarded, and not vice versa, as mentioned above. From (2.18) and (2.19), we finally observe that (3.12) implies $(\gamma_{\hat{1}} - i\gamma_{\hat{2}})\Psi = 0$.

The first requirement (3.9) can be written as

$$(1 - \Gamma^{\hat{r}_1\hat{r}_2})\left[\gamma^{\hat{a}}E^{\hat{a}} + i(\epsilon_{ab}\gamma^{\hat{b}})(\epsilon_{ac}B^{\hat{c}}) + \gamma^{\hat{\psi}}(E^{\hat{\psi}} + iB^{\hat{\psi}})\right]\Psi = 0 \qquad (a = 1, 2,\ \epsilon_{12} = 1)\,. \tag{3.13}$$

The first two terms can annihilate Ψ by choosing $B^{\hat{a}} = \epsilon_{ab}E^{\hat{b}}$, which can be seen from $(\gamma_{\hat{1}} - i\gamma_{\hat{2}})\Psi = 0$. The last term has to be zero by itself, which requires $E^{\hat{\psi}} = B^{\hat{\psi}} = 0$. They can be summarized by a single equation

$$\vec{B} = -I.\vec{E}\,, \tag{3.14}$$

where the vectors are understood to be push-forwarded. Then the second requirement (3.10) is also satisfied. Even after this restriction, we have two real functions as remaining degrees: the magnitude $|\vec{E}| = |\vec{B}|$ and the overall rotation degree of these vectors on the I invariant plane.

We finally comment that the requirement (3.9) is indeed the most general one in the supersymmetry class (2.19). First, one can easily check directly from (3.8) that $B^{\hat{a}} = \epsilon_{ab}E^{\hat{b}}$ has to be imposed: otherwise there cannot be any supersymmetry due to the appearance of matrices like $\gamma^{\hat{\psi}\hat{a}}$. Then, one may keep nonzero E_ψ and B_ψ together with $B_{\hat{a}} = \epsilon_{ab}E_{\hat{b}}$ to solve the supersymmetry condition (3.8) directly. The resulting condition is

$$\frac{1}{\sqrt{(1 + B_{\hat{\psi}}^{\ 2})(1 - E_{\hat{\psi}}^{\ 2})}}\begin{pmatrix} 1 + E_{\hat{\psi}} & -B_{\hat{\psi}}(1 + E_{\hat{\psi}}) \\ -B_{\hat{\psi}}(1 - E_{\hat{\psi}}) & 1 - E_{\hat{\psi}} \end{pmatrix}\begin{pmatrix} R \\ I \end{pmatrix} = \begin{pmatrix} R \\ I \end{pmatrix} \tag{3.15}$$

where R/I denotes the real/imaginary part of the 12 dimensional spinor $\Psi = R + iI$, respectively, in Majorana representation (spinor indices suppressed). This eigenvector equation can be satisfied only when $E_{\hat{\psi}} = B_{\hat{\psi}} = 0$.

3.2 Local (and global) constraints

Apart from the supersymmetry requirement (3.14), we also have to impose the Gauss law constraint and the Bianchi identity: the latter has to be checked also since we have not expressed field strengths in terms of vector potential. To check the Gauss law, we have to compute the electric displacement. The DBI lagrangian is

$$\mathcal{L}_{DBI} = -R^4\sqrt{\det h}\sqrt{1-v^2}\sqrt{1+|\vec{B}|^2-|\vec{E}|^2-(\vec{E}\cdot\vec{B})^2}\,, \tag{3.16}$$

and the Chern-Simon term would not give any contribution. The electric displacement, after imposing the condition (3.10), becomes

$$\Pi^i = \frac{\partial\mathcal{L}}{\partial F_{0i}} = \frac{1}{\sqrt{1-v^2}}\, e^i_{\hat{j}}\, \frac{\partial\mathcal{L}}{\partial E_{\hat{j}}} = R^4\sqrt{\det h}\; e^i_{\hat{j}} E^{\hat{j}}\,. \tag{3.17}$$

The Gauss constraint is

$$\partial_i \Pi^i = 0 \;\; \rightarrow \;\; \partial_i\left(\sqrt{\det h}\; e^i_{\hat{j}} E^{\hat{j}}\right) = 0\,. \tag{3.18}$$

The Bianchi identities become

$$\partial_{[i}F_{jk]} = 0 \rightarrow \partial_i\left(\sqrt{\det h}\, e^i_{\hat{j}} B^{\hat{j}}\right) = 0\,, \tag{3.19}$$

$$\partial_0 F_{ij} + \partial_i F_{j0} + \partial_j F_{0i} = 0 \rightarrow \partial_t\left(\sqrt{\det h}\, e^i_{\hat{j}} B^{\hat{j}}\right) = \epsilon^{ijk}\partial_{[j}\left(\sqrt{1-v^2}\, e^{\hat{l}}_{k]} E_{\hat{l}}\right). \tag{3.20}$$

The second Bianchi identity (3.20) tells us the time evolution of $\vec{B}$ once it is given at initial time. We will not regard it as a constraint: it will be treated as providing time evolution of $\vec{B}$ in the next section.

Here we consider the constraints (3.18) and (3.19) at given time. We plug the vielbein (3.4) and $E^{\hat{\psi}} = 0$ into the two constraints (3.18) and (3.19) to obtain

$$\begin{aligned} \partial_a\left(\sqrt{hg}\, E^{\hat{a}}\right) &= \partial_\psi\left(V_a\sqrt{hg}\, E^{\hat{a}}\right) \\ \partial_a\left(\sqrt{hg}\, B^{\hat{a}}\right) &= \partial_\psi\left(V_a\sqrt{hg}\, B^{\hat{a}}\right), \qquad (a = 1, 2 \text{ summed}) \end{aligned} \tag{3.21}$$

where the caret indices are again the local orthonormal frame ones. Combining the two coordinates x^1 and x^2 into

$$z \equiv x^1 + ix^2\,, \qquad \bar{z} \equiv z^* \;\rightarrow\; \partial \equiv \frac{\partial}{\partial z} = \frac{1}{2}(\partial_1 - i\partial_2)\,, \qquad \bar{\partial} \equiv \frac{\partial}{\partial\bar{z}} = \frac{1}{2}(\partial_1 + i\partial_2) \tag{3.22}$$

the above two constraints are written as

$$\begin{aligned} \partial\left(\sqrt{hg}\,\bar{E}\right) + \bar{\partial}\left(\sqrt{hg}\,E\right) &= \frac{1}{2}\left\{\partial_\psi(\bar{V}\sqrt{hg}\,E) + \partial_\psi(V\sqrt{hg}\,\bar{E})\right\}, \\ \partial\left(\sqrt{hg}\,\bar{B}\right) + \bar{\partial}\left(\sqrt{hg}\,B\right) &= \frac{1}{2}\left\{\partial_\psi(\bar{V}\sqrt{hg}\,B) + \partial_\psi(V\sqrt{hg}\,\bar{B})\right\}, \end{aligned} \tag{3.23}$$

where $E \equiv E^{\hat{1}} - iE^{\hat{2}}$, $B \equiv B^{\hat{1}} - iB^{\hat{2}}$ and $V \equiv V_1 - iV_2$. The supersymmetry requirement (3.14) can be reexpressed as $B = iE$, which allows us to write (3.23) as a single complex equation

$$\bar{\partial}\left(\sqrt{hg}\,E\right) = \frac{1}{2}\,\partial_\psi\left(\bar{V}\sqrt{hg}\,E\right). \tag{3.24}$$

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For the general metric of the form (3.1), it does not look easy to get an explicit solution. Here we will obtain the formal solution of this constraint, but we will also present an explicit solution for the nearly-spherical case in the next section.

We expand the functions $\sqrt{hg}\,E$ and V appearing in (3.24) as ∂_ψ eigenmodes, i.e.,

$$
\begin{aligned}
\sqrt{hg}\,E(z,\bar{z},\psi) &= \sum_{n=-\infty}^{\infty} (\sqrt{hg}\,E)_n(z,\bar{z})\ e^{-in\psi}\,, \\
V(z,\bar{z},\psi) &= \sum_{n=-\infty}^{\infty} V_n(z,\bar{z})\ e^{-in\psi}\,,
\end{aligned} \tag{3.25}
$$

where n runs over a suitable multiple of integers, depending on the ψ period. Then the constraint (3.24) is written as

$$
\bar{\partial}\left(\sqrt{hg}\,E\right)_n = -\frac{\mathrm{in}}{2}\sum_{m=-\infty}^{\infty}(\bar{V})_{n-m}(\sqrt{hg}\,E)_m\,. \tag{3.26}
$$

The formal solution for this equation is

$$
\left(\sqrt{hg}\,E\right)_n = \sum_{m=-\infty}^{\infty} P\exp\left(-\frac{\mathrm{in}}{2}\int d\bar{z}\,\bar{V}\right)_{nm} G_m(z) \tag{3.27}
$$

where $\bar{V}$ is an $\infty\times\infty$ matrix with entry $\bar{V}_{mn}=\bar{V}_{m-n}$, and the expression '$P\exp$' (together with an integral $\int d\bar{z}$) denotes the standard path-ordered product of matrices.

The above formal expression looks messy and not so illuminating. Here we simplify this formal solution for a special case where $\bar{V}$ becomes independent of ψ coordinate. This simplified form will be used to obtain an explicit solution in the next section.[4] In this setting, we only need to consider the mode expansion of $\sqrt{hg}\,E$ in (3.25). Inserting this expansion into (3.27), we get the decoupled expression for each modes

$$
(\sqrt{hg}\,E)_n(z,\bar{z}) = G_n(z)\exp\left(-i\frac{n}{2}\int d\bar{z}\bar{V}(z,\bar{z})\right)\,. \tag{3.28}
$$

The integration in the exponent is an indefinite integral. The holomorphic functions $G_n(z)$ are the integration constants, which are the arbitrary functions surviving the (local) constraints. Note that the $\sqrt{hg}E$ is invariant under the coordinate transformation $\psi\to\psi+f(z,\bar{z})$ compensated by a transformation of V which leaves the metric invariant when h,V,g are independent of ψ.

Note that all the analysis so far does not take any global issues into account. Since the coordinates x^1 and x^2 parametrize a compact 2 manifold, they may develop coordinate singularities at certain points. We should require the solution (3.25) and (3.28) to be well-behaved at these points. In the next section we will give a concrete illustration how to take care of this global constraint, with nearly-spherical giant gravitons as a simple example. Here we present general expectation.

[4]Currently, the only example for this ψ-independent $\bar{V}$ we know is the spherical giant graviton, which will be considered in the next section.

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At coordinate singularities (3.28) may be divergent: divergent solutions are accompanied with unwanted singular sources for the left hand sides of (3.23). We require the function $G_n(z)$ to be sufficiently regular near such coordinate singularities, so as to tame the potential singularities in (3.28) and leave (3.23) source-free. Suppose we chose the coordinate such that there is a coordinate singularity at $z = 0$. Then, discarding the singular modes would truncate the Laurent expansion of $G_n(z)$ into a sort of Taylor expansion. When the 2 manifold has the topology of S^2, as we will study in the next section, there are two coordinate singularities. Two such truncations should be imposed in this case. It would not always be true that there are terms surviving both truncations: there may or may not exist such terms depending on the sign of n in the exponent of (3.28).

There is another form of regularity requirement for E: the energy carried by the gauge field has be finite. We will compute the energy for our BPS configuration in the next subsection, but this criterion should be related to the above source-free condition. We will consider both constraints with the nearly-spherical giant graviton in the next section.

3.3 Energy and angular momenta of the giant graviton

In this subsection, we compute the gauge field contribution to the energy and sum of three SO(6) angular momenta $J_k \equiv J_{x_k y_k}$ $(k = 1, 2, 3)$. These two quantities turn out to be same, showing that the energy carried by the gauge field modes saturates the BPS bound given by the sum of angular momenta.

We first compute the canonical energy. It is given by

$$\mathcal{E} = \dot{\vec{X}} \cdot \frac{\partial \mathcal{L}}{\partial \dot{\vec{X}}} + F_{0i} \frac{\partial \mathcal{L}}{\partial F_{0i}} - \mathcal{L} = R^4 \sqrt{\det h_{ij}} \frac{1 + |\vec{E}|^2}{\sqrt{1 - v^2}} \tag{3.29}$$

after using the supersymmetry conditions. Note that, using the supersymmetry condition $\vec{B} = -I.\vec{E}$, the term depending on gauge field may also be written as $|\vec{E}|^2 = |\vec{E} \times \vec{B}|$.

To calculate the angular momenta, we compute the canonical momenta conjugate to the coordinates $\vec{X}$, which may be regarded as living in $\mathbb{R}^6$. These momenta can be divided into two parts: those coming from the DBI action and from Chern-Simons term. To compute the DBI contribution of the canonical momenta

$$\vec{P}_{DBI} = \frac{\partial \mathcal{L}_{DBI}}{\partial \dot{\vec{X}}} = -\frac{1}{2} \sqrt{-\det(g + F)_{\mu\nu}} [(g + F)^{-1}]^{\mu\nu} \frac{\partial}{\partial \dot{\vec{X}}} [g + F]_{\mu\nu} \,, \tag{3.30}$$

we should first do the $\dot{\vec{X}}$ derivative without fixing the worldvolume gauge like (3.1), and set $\dot{\vec{X}} \cdot \partial_i \vec{X} = 0$ $(i = 1, 2, 3)$ afterward. In the notation of previous subsections, the relevant quantities are given as follows (after imposing the supersymmetry condition (3.14)):

$$\begin{aligned}
\sqrt{-\det(g + F)_{\mu\nu}} &= R^4 \sqrt{\det h} \sqrt{1 - v^2} \,, \\
\left[(g + F)^{-1}\right]^{00} &= -\frac{1}{R^2} \frac{1 + |\vec{B}|^2}{1 - v^2} \,, \\
\left[(g + F)^{-1}\right]^{(0i)} &= \frac{1}{R^2 \sqrt{1 - v^2}} \, e^i_{\hat{j}} (\vec{E} \times \vec{B})^{\hat{j}} \,,
\end{aligned}$$

$$\frac{\partial}{\partial \dot{\vec{X}}}[g+F]_{00} = 2R^2\vec{v}\,, \qquad \frac{\partial}{\partial \dot{\vec{X}}}[g+F]_{0i} = R^2\partial_i\vec{X}\,, \tag{3.31}$$

where the parenthesis on indices means symmetrization. Therefore, we obtain

$$\vec{P}_{DBI} = R^4\sqrt{\det h}\left[(1+|\vec{B}|^2)\frac{\vec{v}}{\sqrt{1-v^2}} - \partial_i\vec{X}e^i_{\hat{j}}(\vec{E}\times\vec{B})^{\hat{j}}\right]. \tag{3.32}$$

The first term is transverse to the 3 manifold Σ, while the second term is longitudinal. Looking at this second term, the vector $\vec{E}\times\vec{B}$ has $\hat{\psi}$ component only. Furthermore, from,

$$\vec{X}_i\, e^i_{\hat{\psi}} = \mathbf{e}^{\psi}\,, \tag{3.33}$$

this longitudinal term is simplified to be

$$-\partial_i\vec{X}\; e^i_{\hat{j}}(\vec{E}\times\vec{B})^{\hat{j}} = -\mathbf{e}^{\psi}(\vec{E}\times\vec{B})^{\hat{\psi}}\,. \tag{3.34}$$

The cross product $(\vec{E}\times\vec{B})^{\hat{\psi}}$ in the second term is simply $-|\vec{B}|^2$, from the supersymmetry requirement (3.14) and the worldvolume orientation $\epsilon_{\psi 12} = 1$ that we chose.

What we need is the sum of three angular momenta, an SO(6) generator corresponding to the rotation $\delta\vec{X} \propto I.\vec{X}$. This is nothing but the velocity vector, decomposed into transverse and longitudinal parts as (3.2). One may rewrite it a bit differently as

$$I.\vec{X} = \vec{v} + \sqrt{1-v^2}\,\mathbf{e}^{\psi}\,. \tag{3.35}$$

The DBI contribution to the sum of three angular momenta is given as

$$[J_1+J_2+J_3]_{DBI} = (I.\vec{X})\cdot\vec{P}_{DBI} = R^4\sqrt{\det h}\,\frac{v^2+|\vec{B}|^2}{\sqrt{1-v^2}}\,, \tag{3.36}$$

which is not the same as the energy (3.29) yet. The contribution from the Chern-Simons term to the sum of three angular momenta is computed in [5], with the solutions without worldvolume gauge fields. We can use that result since, in our background, Chern-Simons term is unchanged after turning on gauge fields. The result is

$$[J_1+J_2+J_3]_{CS} = R^4\sqrt{\det h}\sqrt{1-v^2}\,. \tag{3.37}$$

Adding (3.36) and (3.37), we obtain

$$J_{12}+J_{34}+J_{56} = R^4\sqrt{\det h}\,\frac{1+|\vec{B}|^2}{\sqrt{1-v^2}}\,, \tag{3.38}$$

which is exactly the energy (3.29). Thus we have checked that the energy of giant graviton saturates the BPS bound given by three SO(6) charges even after the gauge fields are turned on.

4. (Nearly-)spherical solutions and quantization

In this section we explicitly construct the gauge field solutions. We will consider nearly-spherical giant gravitons. We will also quantize these modes when the fluctuation is small.

4.1 The explicit solution

The holomorphic surface for a nearly spherical giant graviton having large angular momentum in the X_3-Y_3 plane is given by the equation

$$Z_3 = Z_3^{(0)} + f(Z_1, Z_2)\,, \qquad |Z_1|^2 + |Z_2|^2 + |Z_3|^2 = 1\,, \tag{4.1}$$

where f is a holomorphic function, much smaller than $Z_3^{(0)}$. We also fix the worldvolume diffeomorphism on Σ using a natural parametrization of S^3: we choose the three coordinates α, ϕ_1, ϕ_2 on S^3 (thus on Σ) as follows:

$$\begin{aligned} Z_1 &= \sin\Theta_0 \cos\alpha e^{i\phi_1}\,, \quad Z_2 = \sin\Theta_0 \sin\alpha e^{i\phi_2} \qquad (Z_3^{(0)} \equiv \cos\Theta_0\, e^{i\phi_0} = v\, e^{i\phi_0}) \\ ds^2_{S^3} &= d\alpha^2 + \cos^2\alpha\, d\phi_1^2 + \sin^2\alpha\, d\phi_2^2\,. \end{aligned} \tag{4.2}$$

The ranges of the variables are given as $0 \le \alpha \le \frac{\pi}{2}$ and $\phi_1 \sim \phi_1 + 2\pi$, $\phi_2 \sim \phi_2 + 2\pi$.

We first identify the induced vector field $\sin\alpha\ \mathbf{e}^\psi$, along the direction of which the gauge fields should vanish. It can be easily obtained from (3.2) as

$$\sin\alpha\ \mathbf{e}^\psi = I.\ \mathbf{e}^\perp - \ \mathbf{n}_1 \frac{\mathbf{n}_1 \cdot I.\ \mathbf{e}^\perp}{\mathbf{n}_1 \cdot\ \mathbf{n}_1} - \ \mathbf{n}_2 \frac{\mathbf{n}_2 \cdot I.\ \mathbf{e}^\perp}{\mathbf{n}_2 \cdot\ \mathbf{n}_2} \approx (iZ_1, iZ_2, 0) + \mathcal{O}(f) \tag{4.3}$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are two vectors normal to the holomorphic surface $F(Z_1, Z_2, Z_3) = 0$:

$$\mathbf{n}_1 = (f_1, f_2, -1)\,, \qquad \mathbf{n}_2 = i(f_1, f_2, -1)\,, \qquad \mathbf{n}_1 \cdot\ \mathbf{n}_2 = 0\,. \tag{4.4}$$

Here we used the target $\mathbb{C}^3$ indices like (Z_1, Z_2, Z_3) for the vectors, which are related to the $\mathbb{R}^6$ indices like $Z_k = X_k + iY_k$. To the leading order in f, the vector field (4.3) is $\partial_{\phi_1} + \partial_{\phi_2}$ in our coordinate system (4.2), and this should be proportional to the vector ∂_ψ in the metric (3.3). After doing the following coordinate transformation

$$\psi \equiv \phi_1 + \phi_2\,, \qquad \phi \equiv \phi_1 - \phi_2\,, \qquad \theta \equiv 2\alpha\,, \tag{4.5}$$

we get the S^3 metric in Hopf fibration

$$4ds^2_{S^3} = d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + \cos\theta d\phi)^2 \tag{4.6}$$

with the coordinate range given as

$$0 \le \theta \le \pi\,,\ \phi \sim \phi + 2\pi\,,\ \psi \sim \psi + 4\pi\,. \tag{4.7}$$

Therefore, the base 2 manifold is S^2, having two coordinate singularities at $\theta = 0, \pi$, as mentioned in the previous section.

One can easily obtain the holomorphic coordinates and relevant complex functions from the metric (4.6):

$$z = 2\tan\left(\frac{\theta}{2}\right) e^{i\phi}\,, \quad h = \frac{1}{4}(1-v^2)\,, \quad g = \frac{1}{4}(1-v^2)\left(1 + \frac{z\bar{z}}{4}\right)^{-2}\,, \quad \bar{V} = \frac{i}{\bar{z}}\frac{4 - z\bar{z}}{4 + z\bar{z}}\,. \tag{4.8}$$

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Since ψ is 4π-periodic, n in (3.25) assumes half integer values. The solution for $(\sqrt{hg}\ E)_n$, given by (3.28), is calculated to be

$$\begin{aligned}\exp\left\{-i\frac{n}{2}\int d\bar{z}\bar{V}(z,\bar{z})\right\}G_n(z) &= \left[\frac{z\bar{z}}{(4+z\bar{z})^2}\right]^{n/2} G_n(z)\\ &\sim \left(\sin\frac{\theta}{2}\right)^n\left(\cos\frac{\theta}{2}\right)^n G_n(z)\,, \end{aligned}\tag{4.9}$$

which should be sufficiently regular near $\theta \to 0, \pi$, respectively, in order not to have singular sources there. Let us make a Laurent expansion of $G_n(z)$:

$$G_n(z) = \sum_{k=-\infty}^{\infty} \frac{a^*_{n,k}}{z^k}\,,\tag{4.10}$$

where we included the complex conjugation for the coefficients $a_{n,k}$ for later convenience. The requirement for the 2π periodicity of ϕ_1 and ϕ_2 is that n and k should be either integers or half the odd integers at the same time. Furthermore, forbidding singular sources at $\theta = 0, \pi$, one gets the condition

$$\text{Regularity at } \left\{\begin{array}{l}\theta = 0:\ \ n-k\geq 0\\ \theta=\pi:\ \ n+k\geq 2\end{array}\right\} \to\ -n+2\leq k\leq n\,.\tag{4.11}$$

The total number of modes for given ψ-momentum n is $2n-1$, and the allowed values for n are 1, $\frac{3}{2}$, 2, $\frac{5}{2}$,.... Especially, there are no ψ-independent modes, i.e., $n = 0$. This is natural since the Gauss-Bianchi constraint (3.24) would reduce to that on S^2-base for $n = 0$, which looks too restrictive to admit regular solutions.

To write down the mode expansion, it is more illuminating to advocate a sort of polar basis for the complex fields E and B, given as follows:

$$E^{\text{polar}} \equiv E^{\hat{\theta}} - iE^{\hat{\phi}} = e^{i\phi}(E^{\hat{1}} - iE^{\hat{2}}) = e^{i(\phi_1-\phi_2)}E^{\text{Cart.}}\tag{4.12}$$

where we included the superscript '*Cart.*' to emphasize that complex field we used so far is in cartesian basis. In this polar basis, we have the neat expression for the mode expansion given as

$$\sin\theta\ E^{\text{polar}}(\theta,\phi_1,\phi_2) = \sum_{l_1,l_2=1}^{\infty} a^*_{l_1l_2}\ e^{-il_1\phi_1}e^{-il_2\phi_2}\left(\cos\frac{\theta}{2}\right)^{l_1}\left(\sin\frac{\theta}{2}\right)^{l_2}\tag{4.13}$$

where $l_1 \equiv n+k-1$ and $l_2 \equiv n-k+1$ runs over $1, 2, 3, \ldots$, and $a_{l_1l_2}$'s are complex numbers. This expression will turn out to be the most natural one in the next subsection, in that the angular momenta J_1 and J_2 along the Z_1 and Z_2 plane would be l_1 and l_2, respectively, for each mode.

Note that there is no mode with either of l_1 and l_2 being zero. This is in contrast to the mechanical fluctuation which contains the modes with either of the two angular momenta

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being zero.[5] The electromagnetic fields fall to zero at $\theta = 0, \pi$ (in the orthonomal frame units with caret indices) except for the lowest mode $l_1 = l_2 = 1$ (or $n = 1$)

$$E^{\rm polar}_{1,1}(\theta, \phi_1, \phi_2) = -iB^{\rm polar}_{1,1}(\theta, \phi_1, \phi_2) = a^*_{1,1}\, e^{-i\phi_1} e^{-i\phi_2}\,. \tag{4.14}$$

Together with ∂_ψ, these lowest $E^{\rm polar}_{1,1}$ and $B^{\rm polar}_{1,1}$ form a threesome of nowhere-vanishing orthonormal vector fields on S^3.

Finally, let us check the time evolution of these modes. It is given by the second Bianchi 'identity' (3.20) and the remaining equation of motion

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial F_{\mu i}} \right) = 0 \;\rightarrow\; \partial_t \left(\sqrt{\det h}\, e^i_{\hat{j}} E^{\hat{j}} \right) = -\epsilon^{ijk} \partial_{[j} \left(\sqrt{1 - v^2}\; e^{\hat{l}}_{k]} B_{\hat{l}} \right). \tag{4.15}$$

In general there should be more subtlety since the spatial coordinate frames we have chosen may change by time evolution, thus requiring additional terms due to compensating gauge transformation. However, it does not matter in our spherical case. In this case (4.8), the two equations (3.20) and (4.15) are combined into one holomorphic equation and a real ψ-component equation:

$$\begin{aligned} \partial_t E &= 2i\partial_\psi B = -2\partial_\psi E \\ \partial_t(\sqrt{g} V_a E^{\hat{a}}) &= 2\epsilon_{ab}\partial_a \left(\sqrt{g} B_{\hat{b}}\right) = -2\partial_a \left(\sqrt{g} E_{\hat{a}}\right) \end{aligned} \tag{4.16}$$

where we used $B = iE$ (or $B_{\hat{a}} = \epsilon_{ab} E_{\hat{b}}$) to replace all B's into E's. Then, expressing every field strengths and V with their holomorphic components, we get

$$\begin{aligned} \partial_t E &= -2\partial_\psi E \\ \partial_t(\bar{V} E + V \bar{E}) &= -2 \left(\bar{V} \partial_\psi E + V \partial_\psi \bar{E} \right) \end{aligned} \tag{4.17}$$

where we popped out $\sqrt{g}$ or V_a's from ∂_t since they are all time-independent for the spherical giant case. Inserting the mode expansion $E(z, \bar{z}, \psi) = \sum_n E_n(z, \bar{z}) e^{-in\psi}$, all these equations are solved by giving the time evolution

$$\begin{aligned} \sin\theta\; E^{\rm polar}(z, \bar{z}, \psi, t) &= \sum_{n=1}^{\infty} \sin\theta\; E^{\rm polar}_n(z, \bar{z}) e^{-in(\psi - 2t)} \\ &= \sum_{l_1, l_2 = 1}^{\infty} a^*_{l_1 l_2} \left(\cos\frac{\theta}{2} \right)^{l_1} \left(\sin\frac{\theta}{2} \right)^{l_2} e^{-il_1(\phi_1 - t)} e^{-il_2(\phi_2 - t)}\,. \end{aligned} \tag{4.18}$$

Note that, the phase velocity $\psi/t = 2 = \sqrt{\frac{-g_{tt}}{g_{\psi\psi}}}$ is the light velocity along the ψ direction, measured by the worldvolume metric (3.1) and (4.8).

[5]This can be checked straightforwardly by using the holomorphic surface solutions, keeping the leading contribution of f in (4.1).

4.2 Quantizing the small fluctuations

In this subsection we consider small fluctuation of gauge fields on a (nearly-)spherical giant graviton and quantized them. We will also show that each mode carries integer-valued angular momenta given by l_1 and l_2 identified in (4.18).[6]

We quantize the BPS gauge field modes identified in the previous section assuming the fluctuation is 'small': the meaning of the latter will be addressed more quantitatively as we proceed. To do so, we first compute the DBI lagrangian up to quadratic order in the gauge field strength $F_{\mu\nu}$ (or, equivalently, $\vec{E}$ and $\vec{B}$):

$$\mathcal{L}_{DBI} \cong R^4\sqrt{\det h}\sqrt{1-v^2}\left(-1+\frac{1}{2}|\vec{E}|^2-\frac{1}{2}|\vec{B}|^2+\text{h.o.t.}\right). \tag{4.19}$$

This expansion is valid as long as $|\vec{E}|^2 \ll 1$. This condition will finally be translated into the smallness of occupation numbers after quantization. What we would like to keep is the quadratic term proportional to $|\vec{E}|^2$, which is the kinetic term and should tell us the structure of quantization. We choose the temporal gauge

$$A_0 = 0 \;\rightarrow\; F_{0i} = \dot{A}_i \;(i=1,2,3)\,. \tag{4.20}$$

Since we are only interested in quantizing the BPS modes, we take advantage of the fact $F_{0\psi} = 0$ and hence set $A_\psi = 0$. The quadratic kinetic term can be rewritten in the first order form as follows:

$$\begin{aligned}\frac{R^4}{2}\sqrt{\det h}\sqrt{1-v^2}|\vec{E}|^2 &= \frac{1}{2}\left(R^4\sqrt{\det h}f^i_{\hat{j}}E^{\hat{j}}\right)\left(\sqrt{1-v^2}f_i^{\hat{k}}E_{\hat{k}}\right)\\ &= \Pi^i\dot{A}_i - \frac{\sqrt{1-v^2}}{2R^4\sqrt{\det h}}g_{ij}\Pi^i\Pi^j\,.\end{aligned} \tag{4.21}$$

We will try to do the mode expansion of the first term with the coefficient $a_{l_1l_2}$ defined in (4.18), regarded as off-shell degrees of freedom.

To this end, let us first express the vector potential A_i with this BPS mode expansion. First we re-express the on-shell mode expansion of E as

$$\sin\theta\ E^{\text{polar}} = \frac{\partial}{\partial t}\left[\sum_{l_1,l_2=1}^{\infty}\frac{a^*_{l_1l_2}}{i(l_1+l_2)}e^{i(l_1+l_2)t}\left(\cos\frac{\theta}{2}\right)^{l_1}\left(\sin\frac{\theta}{2}\right)^{l_2}e^{-il_1\phi_1}e^{-il_2\phi_2}\right]. \tag{4.22}$$

Recalling the definition of the vector potential, one gets

$$\sin\theta\ E^{\text{polar}} = e^{i\phi}\frac{\sin\theta}{\sqrt{1-v^2}}\left(f^1_{\hat{1}}\dot{A}_1 - if^2_{\hat{2}}\dot{A}_2\right) = \frac{\sin\theta}{\sqrt{1-v^2}\sqrt{g}}\ \dot{A}^{\text{polar}} \tag{4.23}$$

where we used the complexified vector potential $A^{\text{polar}} = e^{i\phi}A \equiv e^{i\phi}(A_1 - iA_2)$. The series expansion in the square bracket of (4.22) is essentially the expression for A^{polar}, but let us

[6] The quantization of mechanical BPS fluctuation can also be done, using nearly-spherical holomorphic surfaces, and following the procedure of [27–29]. We will not show this result here.

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absorb the on-shell time evolution factor $e^{i(l_1+l_2)t}$ into $a^*_{l_1 l_2}$ as time-dependent $a^*_{l_1 l_2}(t)$ and pretend the off-shell expression

$$A^{\rm polar}(t) = \frac{\sqrt{1-v^2}\sqrt{g}}{\sin\theta} \sum_{l_1,l_2=1}^{\infty} \frac{a^*_{l_1 l_2}(t)}{i(l_1+l_2)} \left(\cos\frac{\theta}{2}\right)^{l_1} \left(\sin\frac{\theta}{2}\right)^{l_2} e^{-il_1\phi_1} e^{-il_2\phi_2} \,. \tag{4.24}$$

One should use this off-shell expression since we are going to read off the quantization rule from the lagrangian defined with the off-shell fields. The term $\Pi^i \dot{A}_i$ containing the information on canonical structure can be re-written as

$$\frac{R^4}{2}\sqrt{hg}\ \bar{E}\dot{A} + ({\rm h.c.}) = \frac{R^4}{2}\sqrt{hg}\ \bar{E}^{\rm polar}\dot{A}^{\rm polar} + ({\rm h.c.})\,, \tag{4.25}$$

where E and $E^{\rm polar}$ are also understood as off-shell expressions, replacing $a^*_{l_1 l_2} e^{i(l_1+l_2)t}$ by $a^*_{l_1 l_2}(t)$. To get the mode expansion of the action, we should also do the integration

$$\int_0^{4\pi} d\psi \int dx^1 dx^2 = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2\, 4\sec^2\frac{\theta}{2}\, \tan\frac{\theta}{2}\,. \tag{4.26}$$

After the integration, the mode expansion for the action is

$$\frac{i}{2}\sum_{l_1,l_2=1}^{\infty} \frac{\pi^2 R^4}{2}(1-v^2)^2 \frac{(l_1-1)!(l_2-1)!}{(l_1+l_2)!}\, a^*_{l_1 l_2}\ \dot{a}_{l_1 l_2} + ({\rm h.c.}) + \cdots \,. \tag{4.27}$$

Therefore, after being promoted to operators, the modes $a_{l_1 l_2}$ satisfy the harmonic oscillator commutation relation in suitable normalization:

$$\left[a_{l_1 l_2}, a^\dagger_{m_1 m_2}\right] = \frac{2}{\pi R^4(1-v^2)^2} \frac{(l_1+l_2)!}{(l_1-1)!(l_2-1)!}\, \delta_{l_1,m_1}\delta_{l_2,m_2}\,. \tag{4.28}$$

The number operators

$$N_{l_1 l_2} = \frac{\pi R^4(1-v^2)^2}{2} \frac{(l_1-1)!(l_2-1)!}{(l_1+l_2)!}\, a^\dagger_{l_1 l_2} a_{l_1 l_2} \tag{4.29}$$

assume integer eigenvalues.

At this point we turn to the question of the 'smallness' of fluctuations, i.e., try to rephrase the criterion $|\vec{E}|^2 \ll 1$ in quantum language. Regarded as an operator with mode expansion (4.18), and also with worldvolume integration, the condition $\int\sqrt{\det h}\ |\vec{E}|^2 \ll \int\sqrt{\det h}$ is given as

$$\sum_{l_1,l_2=1}^{\infty} (l_1+l_2) N_{l_1 l_2} \ll R^4(1-v^2)^2\,. \tag{4.30}$$

We finally compute the quantized angular momentum operators J_1 and J_2's along X_1-Y_1 and X_2-Y_2 planes in terms of the number operators. They are given by

$$J_1 = X^1[P_{DBI}]_{Y_1} - Y^1[P_{DBI}]_{X_1}\,, \qquad J_2 = X^2[P_{DBI}]_{Y_2} - Y^2[P_{DBI}]_{X_2}\,, \tag{4.31}$$

where P_{DBI} is given as (3.32). Inserting

$$(X^1, Y^1) = \sqrt{1-v^2}\cos\frac{\theta}{2}(\cos\phi_1, \sin\phi_1)\,, \qquad (X^2, Y^2) = \sqrt{1-v^2}\sin\frac{\theta}{2}(\cos\phi_2, \sin\phi_2)\,, \tag{4.32}$$

we obtain the following expressions for the angular momentum densities

$$J_1 = R^4\sqrt{1-v^2}\sqrt{hg^2}\cos^2\frac{\theta}{2}|\vec{E}|^2\,, \qquad J_2 = R^4\sqrt{1-v^2}\sqrt{hg^2}\sin^2\frac{\theta}{2}|\vec{E}|^2\,. \tag{4.33}$$

After inserting (4.8), (4.18) and doing the worldvolume integration, the angular momentum operators become

$$J_1 = \sum_{l_1,l_2} l_1 \hat{N}_{l_1 l_2}\,, \qquad J_2 = \sum_{l_1,l_2} l_2 \hat{N}_{l_1 l_2}\,. \tag{4.34}$$

The third angular momentum J_3 along the X_3-Y_3 plane, which should be much larger than the other two in our nearly spherical setting, also carries nonzero contribution from the gauge modes. It is given by

$$J_3 = 2\pi^2 R^4(1-v^2) + \frac{v^2}{1-v^2}\sum_{l_1 l_2=1}^{\infty}(l_1+l_2)\hat{N}_{l_1 l_2} \tag{4.35}$$

where the first and second terms are the contributions from the mechanical part and gauge field fluctuations, respectively. The second term is always much smaller than the first mechanical contribution, taking (4.30) and $v < 1$ into account.

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A. Properties of various vector fields

In this appendix, we briefly recover the properties of various vector fields $\mathbf{e}^{\perp}$, $\mathbf{e}^{\phi}$, $\mathbf{e}^{\psi}$. It is essentially repeating [5], to clarify our convention. We start by a unit vector $\mathbf{e}^{\perp} = (X_k, Y_k)$ perpendicular to S^5. The velocity vector $I.\,\mathbf{e}^{\perp}$ is decomposed into transverse and longitudinal components with respect to the 3 manifold Σ, with the unit vector fields $\mathbf{e}^{\phi}$ and $\mathbf{e}^{\psi}$ defined only on Σ. There is another vector in TS^5 orthogonal to both Σ and $\mathbf{e}^{\phi}$, which we call $\mathbf{e}^{n}$. The action of I on these vectors are given as follows:

$$I.\,\mathbf{e}^{\perp} = \cos\alpha\,\mathbf{e}^{\phi} + \sin\alpha\;\mathbf{e}^{\psi} \tag{A.1}$$

$$I.\,\mathbf{e}^{n} = -\sin\alpha\,\mathbf{e}^{\phi} + \cos\alpha\;\mathbf{e}^{\psi} \tag{A.2}$$

$$I.\,\mathbf{e}^{\phi} = -\cos\alpha\,\mathbf{e}^{\perp} + \sin\alpha\;\mathbf{e}^{n} \tag{A.3}$$

$$I.\,\mathbf{e}^{\psi} = -\sin\alpha\,\mathbf{e}^{\perp} - \cos\alpha\;\mathbf{e}^{n}\,. \tag{A.4}$$

The first one just follows from the definition of $\mathbf{e}^{\psi}$ and $T_0\Sigma$. We set the velocity $v = \cos\alpha$ to be positive, and also set $\sin\alpha$ to be positive, which is the convention for $\mathbf{e}^{\psi}$. We will derive the remaining three relations, trying to distinguish the derived facts and conventions.

Recall that there is a sub-plane $T_0\Sigma$ of $T\Sigma$ which is invariant under the action of I. In fact, such pair of directions in Σ is guaranteed to exist from the fact that it is constructed from the intersection of a holomorphic surface and S^5. Since $\mathbf{e}^\perp$ is orthogonal to $T_0\Sigma$, so is $I.\ \mathbf{e}^\perp$. Otherwise $T_0\Sigma$ would not be invariant under I. Therefore, $\mathbf{e}^\psi$ is orthogonal to $T_0\Sigma$ from its definition (A.1). We have four unit vector fields $\mathbf{e}^\perp$, $\mathbf{e}^\phi$, $\mathbf{e}^n$ and $\mathbf{e}^\psi$ orthogonal to $T_0\Sigma$, which should also close under I.

First, $I.\ \mathbf{e}^\phi$ is expanded by $\mathbf{e}^\perp$, $\mathbf{e}^n$ and $\mathbf{e}^\psi$. But it should be orthogonal to $\mathbf{e}^\psi$ from (A.1): If $\mathbf{e}^\phi$ and $\mathbf{e}^\psi$ had components which mixes into each other by I, acting one more I on (A.1) would still yield some component tangent to S^5 in the right hand side, which is a contradiction. So we have $I.\ \mathbf{e}^\phi = A\ \mathbf{e}^\perp + B\ \mathbf{e}^n$. Taking the norm of this with $\mathbf{e}^\perp$, we obtain $A = -\cos\alpha$. This proves the relation (A.3), where the $+$ sign of B is our convention for the $\mathbf{e}^n$ direction. Then (A.2) and (A.4) are obtained from (A.1) and (A.3) by applying the complex structure I.

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